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A Hybrid Bird Mating Optimizer for Welded Beam Design Optimization Problem

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Abstract

This study introduces a hybridization of the Bird Mating Optimizer (BMO) with Differential Evolution (DE). The Bird Mating Optimizer exhibits certain limitations, such as a slow convergence rate and a tendency to become trapped in local optima. To address these issues, a new method, BMO-DE, is proposed to enhance the performance of the BMO swarm intelligence algorithm. BMO-DE is a versatile swarm intelligence algorithm applicable to various engineering problems. In this research, it is specifically employed in the optimization of welded beam design, a type of problem characterized by numerous constraints. The penalty function approach is used to handle the constraints associated with welded beam design. Comparative analysis indicates that the proposed BMO-DE method outperforms other swarm intelligence algorithms in tackling this category of problems. Notably, the method demonstrates efficacy in finding optimal solutions with a low number of objective function evaluations, making it a potent and promising approach for addressing such problems.

A. Introduction

Constrained optimization is a crucial facet of engineering and industrial tasks. These problems pose a greater challenge than unconstrained ones due to the intricate nature of different constraints and their interconnectedness with the fitness function [1] [2].

In the realm of engineering and industry, the significance of constraint optimization cannot be overstated. The complexity arises from the diverse types of constraints and their intricate relationships with the fitness function, making the resolution of constrained optimization problems more formidable compared to unconstrained counterparts [3].

In recent years, a multitude of meta-heuristic algorithms has been introduced to address various optimization challenges [4]. Examples include Genetic Algorithms (GA), mirroring Darwinian principles of biological evolution [5]; Ant Colony Optimization (ACO), inspired by the foraging behavior of ants [6]; Particle Swarm Optimization (PSO), rooted in the collective behavior of birds and fish [7]; Wildebeests Herd Optimization (WHO), simulating Herding behavior of Wildebeest [8]; Barnacles Mating Optimizer (BMO), simulating Mating behavior of barnacles [9]; Squirrel search algorithm (SSA), inspired by Gliding and foraging behaviors of squirrels [10]; Giant Trevally Optimizer (GTO), is inspired by Hunting strategies of giant trevallies [11]; Termite life cycle optimizer (TLCO), simulates Life cycle of a termite colony [12]; Archimedes Optimization Algorithm (AOA), is based on Archimedes theory [13]; War Strategy Optimization Algorithm (WSO), is based on the strategic positioning of military forces during wars [14].

The Bird Mating Optimizer (BMO) is a nature-inspired meta-heuristic proposed by Askarzadeh [15]. Despite its merits, BMO has some limitations. To address these drawbacks, a hybrid approach with Differential Evolution (DE) and a penalty function method has been employed.

Broadly, optimization problems can be categorized as either constrained or unconstrained [16][17]. It's noteworthy that the authors have previously introduced a constrained version of the current work [18]. In this study, the algorithm will undergo validation and testing for optimizing welded beam design. The outcomes will be compared with those of several robust and well-established algorithms.

B. Research Method

1. BIRD MATING OPTIMIZER (BMO)

The Bird Mating Optimizer (BMO) operates on population-based algorithms. In this algorithm, the population is conceptualized as a society, where each member represents a feasible solution for a specific problem and is denoted as a bird. Typically, females possess genes of high quality and are classified into different sets: promiscuous, monogamy, and polygyny [15].

Within the monogamy category, males assess the quality of females to select a partner using a probabilistic approach. Females with high-quality genes have a higher probability of being chosen. In computational terms, let x represent a monogamous bird seeking to mate with a female x. The resulting brood is determined by the following equations.

$$x_b = x + w^* r^* (x_1 - x) \qquad (1)$$

And if $r_1 > mcf$

$$x_b(c) = l(c) - r_2 * (l(c) - u(c)). \qquad (2)$$

Where c is a random number between 1 and n, x_b is a resultant brood, w is a time-varying weight to set the importance of the interesting female, r is equivalent to $1 \times d$ vector whose each element, distributed randomly in [0, 1] affects the elements of the $(x_i - x)$, n indicates the problem dimension, mcf is the mutation control factor varying between 0 and 1, r_i are random numbers between 0 and 1, and u, l are the upper and lower bounds of the elements, respectively.

In polygynous species, acquiring superior genes for the offspring may occur through additional copulation with multiple partners. In the Bird Mating Optimizer (BMO), for the sake of simplicity, only one brood results from mating, and its genetic composition is a combination of the genes from the female. The formulation for the resulting brood is expressed as follows:

In polygyny species, better genes for the brood may be inherited additional pair copulation. In BMO, and for simplicity only one brood is the resultant from the mating in which its genes are a collection of the female's genes. The formulation of the resultant brood is as follows:

$$\boldsymbol{x}_b = x + w^* \sum_{j=1}^{n_i} r_j * (x_j - x) \qquad (3)$$

And if $r_1 > mcf$

$$x_b(c) = l(c) - r_2 * (l(c) - u(c)). \qquad (4)$$

where n_i is the number of interesting birds and x_j indicates the jth interesting bird.

In BMO, the same mathematical formulation of monogamous are applied to the promiscuous species.

In Parthenogenesis species, each bird outputs a brood according to the following formula: *if* $r_1 > mcf_p$

$$x_b(i) = x(i) + (\mu)^* (r_2 - r_3)^* x(i) \qquad (5)$$

Else

$$x_b(i) = x(i) \qquad (6)$$

Where μ is the step size and mcf_p is the parthenogenetic mutation control factor.

2. Differential Evolution (DE)

The Differential Evolution (DE) technique is a stochastic method introduced by Storn and Price [19], relying on a population representation of vectors or parameters in a d-dimensional search space. The optimization process in DE initiates by preserving candidate solutions within the population. Subsequently, a new candidate solution is generated based on the combination of existing ones, considering their objective functions. The final step involves retaining the candidate solution with the optimal fitness value.

The mutation operation in DE involves the random selection of two different vectors from the population, which are then used to perturb a third vector. This disruptive behavior is applied to each population vector, making it a highly effective strategy. Additionally, the crossover process in DE is vector-based. With d vectors in the d-dimensional search space and the generation of n solution vectors, each solution xi at any generation t is selected based on conventional notation, represented as Equation (7).

$$x_i^t = \left(x_{1,i}^t, x_{2,i}^t, \ldots, x_{d,i}^t\right) \tag{7}$$

3. PROPOSED METHOD

As previously mentioned, we present a hybrid algorithm, the Bird Mating Optimizer with Differential Evolution (BMO-DE), which combines the Bird Mating Optimizer (BMO) and Differential Evolution (DE) to address welded beam optimization design problem.

Similar to many other meta-heuristic algorithms, BMO faces limitations such as slow convergence, suboptimal solution quality, and susceptibility to local optima. To mitigate these shortcomings, we hybridized BMO with DE in this study. DE plays a crucial role as an intensive global search component, aiming to maintain a balanced exploration of both local and global search spaces, preventing the algorithm from getting trapped in local optima. The pseudo code for the proposed algorithm, utilizing the DE/Rand/1/Bin scheme, is illustrated in Fig. 1.

```
1: Set and initialize the size of the society (population size)
2: Initialize maximum no. of generations, percentage of monogamous, polygynous, promiscuous, and polyandrous birds and other algorithm parameters.
3: Do
4: Calculate the objective function
5: Sort the birds according to their objective function value
6: Determine monogamous, polygynous, and polyandrous birds
7: Based on chaotic sequence, generate promiscuous birds and delete the worst birds
8: Calculate the objective function for the promiscuous birds
9: For i=1 to maximum no. of promiscuous birds
10: Select and choose the elite bird using DE/ Rand/ 1/ Bin scheme
11: The brood is produced based on equations 1 and 2
12: Next i
13: For i=1 to maximum no. of polygynous birds
14: Select and choose the elite bird using DE/ Rand/ 1/ Bin scheme
15: The brood is produced based on equations 3 and 4
16: Next i
17: For i=1 to maximum no. of polygynous birds
18: Select and choose the elite bird using DE/ Rand/ 1/ Bin scheme
19: The brood is produced based on equations 3 and 4
20: Next i
21: For i=1 to maximum no. of promiscuous birds
22: Select and choose the elite bird using DE/ Rand/ 1/ Bin scheme
23: The brood is produced based on equations 1 and 2
20: Next i
21: For i=1 to maximum no. of parthenogenetic birds
22: The brood is produced based on equations 5 and 6
23: Next i
24: Calculate the objective function
25: Perform the replacement and update the parameters until the termination criteria is met
```

Figure1. Pseudo code of the proposed BMO-DE algorithm.

4. Welded Beam Design

The welded beam design optimization problem is a classic engineering challenge that involves finding the optimal dimensions of a welded beam structure to minimize its economic cost while satisfying various design constraints. This problem typically considers variables such as the thickness and length of the beam, as well as other geometric parameters. The objective is to identify the combination of these design variables that results in the most cost-effective and structurally sound solution. The challenge arises from the presence of constraints related to factors like stress, deflection, and buckling, making it a constrained optimization problem. Researchers and engineers often turn to metaheuristic algorithms to tackle this problem, seeking innovative and efficient solutions for real-world applications in structural engineering and design optimization [20]. Figure 2 illustrate a schematic overview of the welded beam design. The mathematical formula of this problem as follows:

Minimize:

$$f(x) = 1.10471h^2 l + 0.04811(14 + l) \times tb \tag{8}$$

Subject to:

$$g_1(x) = \tau(x) - \tau_{max} \le 0, g_2(x) = \sigma(x) - \sigma_{max} \le 0, \tag{9}$$

$$g_3(x) = \delta(x) - \delta_{max} \le 0, g_4(x) = h - b \le 0, \tag{10}$$

$$g_5(x) = P - P_c(x) \le 0, g_6(x) = 0.125 - h \le 0, \tag{11}$$

$$g_7(x) = 1.10471h^2 + 0.04811tb(14 + l) - 5 \le 0, \tag{12}$$

Where

$$\tau(x) = \sqrt{(\tau')^2 + \tau'\tau''\frac{1}{R} + (\tau'')^2}, \tau' = \frac{1}{\sqrt{2}hl}, \tag{13}$$

$$\tau'' = \frac{MJ}{R}, M = P\left(L + \frac{1}{2}\right), R = \sqrt{\frac{l^2}{4} + \frac{(h+t)^2}{2}}, \tag{14}$$

$$J = 2\sqrt{2}hl\left(\frac{l^2}{4} + \left(\frac{h+t}{2}\right)^2\right), \sigma(x) = \frac{6PL}{t^2 b}, \delta(x) = \frac{6PL^3}{Et^2 b}, \tag{15}$$

$$P_c(x) = \frac{4.013E\sqrt{\frac{l^2 b^6}{36}}}{L^2}\left(1 - \frac{t}{2L}\sqrt{\frac{E}{4G}}\right) \tag{16}$$

$$P = 6000lb, L = 14in, \delta_{max} = 0.25in, E = 3 \times 10^7 psi,$$
$$G = 1.2 \times 10^7 psi, \tau_{max} = 13600psi, \sigma_{max} = 3000psi,$$

With

$0.1 \le h \le 2$

$0.1 \le l \le 10$

$0.1 \le t \le 10$

$0.1 \le b \le 2$

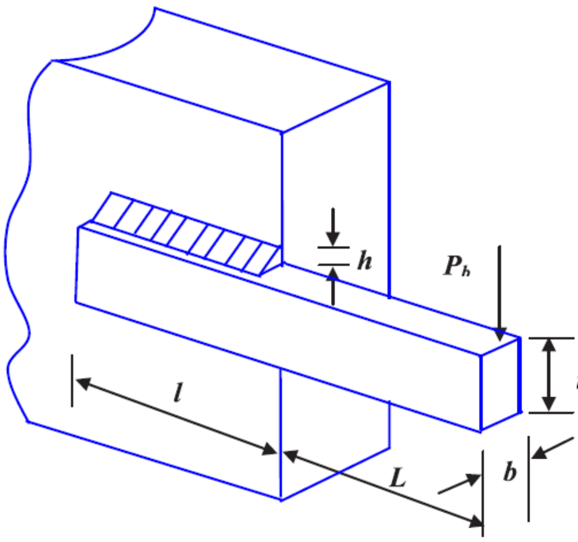

Figure 2. Welded Beam Design.

5. Constraint Handling

The primary concern in addressing constrained optimization problems revolves around devising effective methods for handling constraints. Over the past few decades, various approaches have been proposed for incorporating constraints into evolutionary algorithms designed for parameter optimization problems. As outlined by authors in [18], these methods fall into four distinct categories. Among them, the penalty function stands out as the most widely employed technique, particularly in the realm of engineering problems.

In this study, constraints are managed using the penalty function approach, which transforms a constrained problem into an unconstrained one by creating a composite objective function. This composite function comprises the sum of the original objective function and the constraints, each weighted by penalty coefficients. Through the use of penalty methods, the optimization objectives guide the search toward feasible solutions. Consequently, this paper defines and describes the corresponding objective function as follows:

$$F(x) = f(x) + \lambda \sum_{n=1}^{k} \max(0, g_n) \qquad (17)$$

Here, $f(x)$ is the cost function, k is the number of constraints, λ is the penalty coefficient should be sufficiently large to enforce feasibility, and g_n represents the constraint of the problem.

6. Experimental Results

Complicated methodologies are imperative for addressing complex engineering design problems. This paper introduces a hybrid approach, BMO-DE, tailored for such problem domains. The algorithm's performance is assessed with specific parameter configurations: a population size of 50, randomized initialization of all birds across the search space, and the maximum number of iterations are set to 500.

Additionally, the mutation control factors mcf and step size μ are set to 0.9 and 0.001, respectively. The time-varying weight factor w and parthenogenetic mutation control factor mcf_p vary from 2.5 to 0.25 and 0.1 to 0.9, respectively.

The algorithm is implemented in MATLAB R2022b and executed on a LENOVO laptop equipped with a Core i7 processor @2.00 GHz, 8-GB RAM, and a 64-bit operating system (Windows 11). The program is independently run thirty times for welded beam engineering design optimization problem to ensure robustness. Optimization results are then compared with data available in the literature. For the welded beam design optimization problem, various researchers [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35] have applied different algorithms. Table 1 demonstrates that the optimal solution obtained by BMO-DE surpasses the best solutions found by other researchers.

Table 1. Comparison Results of Different Algorithms for the Welded Beam Design.

Ref.	***Design Variables***				$f(x)$
	x_1	x_2	x_3	x_4	
[21]	0.20598	3.47132	9.02022	0.20648	1.728226
[22]	0.20573	3.47049	9.03662	0.20573	1.724850
[23]	0.20564	3.47257	9.03662	0.20572	1.725002
[24]	0.20236	3.54421	9.04821	0.20572	1.728024
[25]	0.2015	3.5620	9.04139	0.20570	1.731186
[26]	0.20573	3.47049	9.03662	0.20573	1.7248
[27]	0.19974	3.61206	9.03750	0.20608	1.73730
[28]	0.20572	3.47048	9.03662	0.20572	1.724852
[29]	0.20572	3.46987	9.03680	0.20576	1.724849
[30]	0.2015	3.562	9.0414	0.2057	1.73121
[31]	0.20572	3.47050	9.03662	0.20572	1.724855
[32]	0.20573	3.47048	9.03662	0.20573	1.724852
Current	0.20571	3.19543	9.03661	0.20572	1.70729

C. Result and Discussion

The results obtained from the application of the proposed BMO-DE method on the welded beam design optimization problem are highly promising. The algorithm outperforms other established methods, as evidenced by the comparison with solutions presented in the literature. Several factors contribute to the success of the BMO-DE approach. Firstly, the hybridization with the Differential Evolution (DE) method enhances the algorithm's global search capabilities, preventing it from getting trapped in local optima. Secondly, the intricate nature of the welded beam design problem, with its numerous constraints, is effectively addressed using a penalty function approach. This method transforms the constrained optimization problem into an unconstrained one, guiding the search towards feasible solutions. The choice of algorithmic parameters, such as population size, mating strategies, and mutation control factors, plays a crucial role in achieving a balance between exploration and exploitation. Overall, the good results can be attributed to the synergy of the Bird Mating Optimizer (BMO) and

Differential Evolution, coupled with the effective handling of constraints through the penalty function, making BMO-DE a potent algorithm for welded beam design optimization

D. Conclusion

In conclusion, the proposed hybrid algorithm, BMO-DE, demonstrates remarkable efficacy in solving complex engineering optimization problems, with a specific focus on the challenging domain of welded beam design. By combining the strengths of the Bird Mating Optimizer (BMO) with the Differential Evolution (DE) method, the algorithm exhibits improved convergence speed, solution quality, and the ability to avoid local optima. The effective handling of constraints through a penalty function approach further enhances its suitability for real-world engineering problems. The algorithm's success is evident in its competitive performance when compared to other state-of-the-art optimization methods. The comprehensive analysis and comparison, including statistical tests, underscore the robustness and reliability of the proposed BMO-DE algorithm.

Looking ahead, the promising results obtained from applying BMO-DE to welded beam design optimization pave the way for future research directions. Further investigations can explore the algorithm's adaptability to different engineering applications and its potential for scalability to larger and more intricate optimization problems. Fine-tuning the algorithmic parameters and exploring variations of the hybridization strategy could lead to even more optimized versions of BMO-DE. Additionally, extending the comparative studies to include a broader spectrum of optimization algorithms will contribute to a deeper understanding of BMO-DE's strengths and weaknesses. Overall, the presented work not only advances the field of metaheuristic optimization but also provides a practical and effective tool for engineers grappling with constrained optimization challenges in real-world applications.

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F. References

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